[2][a] $a_{n+1} - a_n = \frac{6^{n+1}}{(2(n+1)+1)!} - \frac{6^n}{(2n+1)!}$

$$= \boxed{\frac{6^{n+1}}{(2n+3)!} - \frac{6^n}{(2n+1)!}} \cdot \boxed{\frac{(2n+3)(2n+2)}{(2n+3)(2n+2)}}$$

1 1

$$= \boxed{\frac{6^{n+1} - 6^n(2n+3)(2n+2)}{(2n+3)!}}$$

1

$$= \frac{6^n[6-(4n^2+10n+6)]}{(2n+3)!}$$

$$= \boxed{\frac{-6^n(4n^2+10n)}{(2n+3)!}} \boxed{< 0} \longrightarrow a_{n+1} < a_n$$

2 $\frac{1}{2}$

so $\{a_n\}$ is

$\frac{1}{2}$ DECREASING &

$\frac{1}{2}$ MONOTONIC

[b] $$\left\{\frac{n-6}{2n-7}\right\} = \left\{\frac{-5}{-5}, \frac{-4}{-3}, \frac{-3}{-1}, \frac{-2}{1}, \frac{-1}{3}, \dots\right\}$$

$$\overset{1}{=} \left\{\boxed{1, \frac{4}{3}, 3, -2}, -\frac{1}{3}, \dots\right\}$$

$\frac{1}{2}$ $\boxed{a_1 < a_2}$ BUT $\boxed{a_3 > a_4}$ $\frac{1}{2}$

SO $\{a_n\}$ IS $\boxed{\text{NOT MONOTONIC}}$ $\frac{1}{2}$

[3][a] $a_n = \dfrac{2n^2 + n^{\frac{3}{7}}}{3n^2 + n^{\frac{7}{3}}}$ 1 $b_n = \dfrac{2n^2}{n^{\frac{7}{3}}} = \dfrac{2}{n^{\frac{1}{3}}}$

$\frac{1}{2}$ $a_n, b_n > 0$ FOR $n \geq 1$

1 $\displaystyle \lim_{n\to\infty} \frac{2n^2 + n^{\frac{3}{7}}}{3n^2 + n^{\frac{7}{3}}} \cdot \frac{n^{\frac{7}{3}}}{2n^2} = \lim_{n\to\infty} \frac{2n^2 + n^{\frac{3}{7}}}{2n^2} \cdot \frac{n^{\frac{7}{3}}}{3n^2 + n^{\frac{7}{3}}}$ $\frac{1}{2}$

$\displaystyle = \lim_{n\to\infty} \frac{2 + \frac{1}{n^{\frac{11}{7}}}}{2} \cdot \frac{1}{\frac{3}{n^{\frac{1}{3}}} + 1}$ 1

$\frac{1}{2}$ $= 1 \neq 0$ $\frac{1}{2}$

$\frac{1}{2}$ $\displaystyle 2 \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{3}}}$ DIV 1 (p-SERIES $\frac{1}{2}$ $p = \frac{1}{3}$ $\frac{1}{2}$ ≤ 1)

SO $\displaystyle \sum_{n=1}^{\infty} \frac{2n^2 + n^{\frac{3}{7}}}{3n^2 + n^{\frac{7}{3}}}$ DIV $\frac{1}{2}$ (LIMIT COMPARISON) 1

[b] $\lim\limits_{n\to\infty}\left(2-e^{\frac{1}{n}}\right)^n = \lim\limits_{n\to\infty} e^{n\ln\left(2-e^{\frac{1}{n}}\right)} = \boxed{e^{\lim\limits_{n\to\infty} n\ln\left(2-e^{\frac{1}{n}}\right)}} = \boxed{e^{-1}}$ 1 1

$\boxed{\neq 0}$ $\frac{1}{2}$

SO, $\boxed{\begin{array}{l}\sum\limits_{n=1}^{\infty}\left(2-e^{\frac{1}{n}}\right)^n \\ \text{DIV}\end{array}}$ $\frac{1}{2}$

$\left(\boxed{\text{DIVERGENCE}}\right)$ 1

$\lim\limits_{x\to\infty} x\ln\left(2-e^{\frac{1}{x}}\right) = \boxed{\lim\limits_{x\to\infty}\dfrac{\ln\left(2-e^{\frac{1}{x}}\right)}{\frac{1}{x}}}$ 1

$= \boxed{\lim\limits_{x\to\infty}\dfrac{\frac{1}{2-e^{\frac{1}{x}}}\left(-e^{\frac{1}{x}}\right)\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}}$ 2

$= \boxed{\lim\limits_{x\to\infty}\dfrac{-e^{\frac{1}{x}}}{2-e^{\frac{1}{x}}}}$ 1

$= \dfrac{-e^0}{2-e^0}$

$= \boxed{\dfrac{-1}{2-1}}$ 1

$= \boxed{-1}$ $\frac{1}{2}$

[c] $-1 \le \cos n \le 1$ $\frac{1}{2}$

$4 \geq -4\cos n \geq -4$ $\frac{1}{2}$

$11 \geq 7-4\cos n \geq 3$

$3 \le 7-4\cos n \le 11$ 1

$0 < 3e^{-n} \le (7-4\cos n)e^{-n} \le 11e^{-n}$ 1

$\frac{1}{2}$

$0 \le (7-4\cos n)e^{-n} \le 11\left(\frac{1}{e}\right)^n$

1 $\frac{1}{2}$ $\frac{1}{2}$

$\frac{1}{2}$ $11\sum_{n=1}^{\infty}\left(\frac{1}{e}\right)^n$ CONV (GEOMETRIC $|r| = \frac{1}{e} < 1$)

$\approx \frac{1}{3}$

SO $\sum_{n=1}^{\infty}(7-4\cos n)e^{-n}$ CONV (COMPARISON)

$\frac{1}{2}$ 1